**The Time Value of Money Part 2 – Chapter 4 in RWJJ**

Interest Rate is 5%. What is the total PV of: $1,000 at end of year 1,2,3,4 &5?

PV = $1,000 + $1,000 + $1,000 + $1,000 + $1,000

1.05 1.052 1.053 1.054 1.055

= 952.38 + 907.03 + 863.84 + 822.70 + 783.53

= $4,329.48

Excel: Find PV on function wizard

Rate = .05

Nper = 5

Pmt = 1,000

Fv = blank

Type = blank

**Here’s another way to look at it:**

$4,329.48 will provide $1,000/year for 5 years if the interest rate is 5%.

Year Start Interest Earned Total Payout Remainder

1 $4,329.48 $216.47 $4545.95 $1000 $3545.95

2 $3,545.95 177.30 3723.25 1000 2723.25

3 $2,723.25 136.16 2859.41 1000 1859.41

4 $1,859.41 92.97 1952.38 1000 952.38

5 $ 952.38 47.62 1000.00 1000 0

So, at 5%, how much do we need to provide $1,000/year **forever**?

If it’s to go forever, we can’t use up any principal, so, we must produce $1,000 in interest each year.

# At 5%, how much principal is needed to produce $1,000 in interest?

Interest = Interest Rate x Principal

Principal = \_\_\_Interest\_\_\_ = $1,000 = $20,000

Interest Rate .05

So $20,000 is the principal needed to produce $1,000 interest at 5%

$20,000 · 5% = $1,000 - Pay the $1,000 and start over each year.

**Perpetuity** - A stream of level cash flows that never ends.

PV of Perpetuity = Cash Flow or, PVP = C

Int. Rate r

**Annuity** - Stream of level cash payments that ends. It’s not an annuity if the payments are not all equal.

$1 million lottery is an annuity of $50,000/year for 20 years.

Earlier example was an annuity: $1,000/year for 5 years. Remember what the PV was?

PV = $1,000 + $1,000 + $1,000 + $1,000 + $1,000 = $4,329.48

1.05 1.052 1.053 1.054 1.055

There is a quicker way:

Year: 1 2 3 4 5 6 7 8 ...........

Perpetuity A $1 $1 $1 $1 $1 $1 $1 $1 ...........

Perpetuity B $1 $1 $1 ...........

A 5 year annuity = Perpetuity A - Perpetuity B

Perpetuity A = 1 Perpetuity B = 1 · \_\_\_1\_\_\_ = value at the

r r (1+r)5 beginning of yr. 5

discounted to today

PVA =  The expression in brackets is called the annuity factor.

From our previous example:PVA = 

= $4,329.48 which is the same result we got previously

Note that in an ordinary annuity, the present value is the value at one compounding period **before** the first cash flow. For example – if the first cash flow is at time period 1, the PV of an ordinary annuity will give you the value at time zero.

Example: State lottery: How much is $50,000/year for 20 years really worth?

Assume a 7% interest rate.

PVA =  = 

Excel: Nper = 20

Rate = 7

PMT = -50,000

Solve for PV = 529,700.71

But, in the lottery, you get your first payment immediately and then the 19 others. In other words, you get each payment at the **beginning** of the year, rather than (as we usually do) at the end of the year. This is called an **Annuity Due**.

With your calculator, just put it in the “Begin” mode.

With Excel, enter “1” for type.

PVAD = $50,000 + 

= 50,000 + 516,779.76 = 566,779.76

Also: Annuity Due = Ordinary Annuity (1+r)

$529,700.71 (1.07) = $566,779.76

PVAD = C  or  (1+r)

Note that it’s higher than when you get payments at the end of each year.

**Home Mortgages**: **Example**:

Amount of the Mortgage = $200,000

Obtain a 30 year mortgage at 6% APR

Monthly Payments

PVA = 

$200,000 =   = 

C = $1,199.10 = monthly mortgage payment

In Excel, you are solving for the payment.

End of first month: Original Principal = $200,000

Interest due = 6%/12 = .5% = .005

.005 · $200,000 = $1,000

You pay $1,199.10

Interest 1,000.00

Princ. $ 199.10

End of second month: New Princ. = $200,000 - $199.10 = $199,800.90

Int. Due = .005 x $199,800.90 = $999.00

You pay $1,199.10

Interest 999.00

Princ. $ 200.10

For 30 years, each month, the interest decreases and the principle increases. On the last month, you pay off all the principle with only a few dollars of interest.

Interest is due on the unpaid balance each month - not on the entire $200,000.

This is called **amortizing a loan**. Some money is paying the principal and some is paying the interest.

Anytime we want to know what the payoff of our loan is, we merely have to look at the present value of the remaining future payments.

How much do we still owe on the loan after 10 years of payments (20 years remaining)?

PVA = 

= $167,371.45

**A Delayed Annuity**

What is the PV of $1,000 to be received each year for 10 years, starting five years from now if the discount rate is 7%?

1000--------------------------------------------------------1000

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

PV = 

= 7,023.58

(1.07)4

= 5,358.26

Note that you have to do this in two steps with Excel. First find the PV of the annuity as of time 4 (7,023.58). Then find the PV of that lump sum at time zero. Whenever you do a two-step problem like this in Excel, you should always **reference the cell** with the number you want to input – don’t write it down and then type it in or you will have rounding errors.

## Future Value of an Annuity

FVA = 

How did we derive that formula?

FV = PV (1+r)t

Substitute PV of an annuity in for PV above:

FV =  (1+r)t

= 

= 

**Example**:

Invest $1,000 each month into my 401(k) plan at work. If we assume that our return will be 6% per year (APR), how much will be in our 401(k) after 35 years?

Note that 6% APR means 0.5% per month (6%/12 = 0.5%) and 35 years means 420 months.

FVA = $1,000  = $1,424,710.30

**Combining PV and FV in one Problem**

How much do you need to save each year for 10 years, so that you can pay for 4 years of college at $30,000 per year with an interest (investment) rate of 6%?

30 30 30 30

\_\_\_\_\_C\_\_\_C\_\_\_C\_\_\_C\_\_C\_\_\_C\_\_\_C\_\_\_C\_\_C\_\_\_C\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

0 1 2 3 4 5 6 7 8 9 10 11 12 13

PVAD = $(1.06)

= $110,190.36 at year 10

$110,190.36 = C 

C = $ 8,359.92

**Present Value of a Growing Perpetuity**

Suppose you are getting a dividend payment of $100 per year and you expect the company’s dividends to grow at the rate of 3% per yer forever. What is the present value of this growing perpetuity if the appropriate interest rate is 12%?

PVGP = \_\_C1\_\_

r-g

PVGP = 100

.12-.03

PVGP = 1111.11

Note that if g = 0, we have our standard perpetuity formula.