**The Time Value of Money Part 1 – Chapter 4 in RWJJ**

The time value of money - The most basic concept in Finance.

**Definitions:**

**Interest** - Money paid for the use of your money. Expressed as a % or a decimal.

**Future Value** - Amount to which an investment will grow after earning interest.

**Present Value** – Amount of money you start with – the initial investment.

**Example**: Interest = Interest Rate x Present Value (your initial investment)

 $6 = .06 · $100

Value after one year = $100 + $6 = $106

Let r = interest rate; PV = Present Value and FV = Future Value

Value after one year = PV (1+r)

 $100 (1 + .06) = $106

Second Year - Start with $106

 Interest = $106 ·.06 = $6.36

Value at end of year = $106 + $6.36 = $112.36

 = $106 (1 + r)

 = $106 (1 + .06) = $112.36

Start at beginning to year one and go to the value at the end of year 2.

100\_\_\_\_\_\_\_\_\_\_\_106\_\_\_\_\_\_\_\_\_\_\_\_\_112.36

 0 1 2

Value after 2 years = $100 (1.06) (1.06) = $112.36

 = $100 (1.06)2 = $112.36

 = PV (1+r) (1+r)

 = PV (1+r)2

FV = PV (1+r)t

where t = number of compounding periods

and r = interest rate per compounding period

Taking it out to 5 years:

100\_\_\_\_\_\_106\_\_\_\_\_\_\_112.36\_\_\_\_\_\_\_119.10\_\_\_\_\_\_\_\_126.25\_\_\_\_\_\_\_133.82

 0 1 2 3 4 5

FV = PV (1 + r)t Here r = .06 = 6%

for t = 3 FV = 100 (1.06)3 = $119.10

for t = 4 FV = 100 (1.06)4 = $126.25

for t = 5 FV = 100 (1.06)5 = $133.82

On **Excel**:

Locate Future Value with the Function Wizard

1. Enter .06 or 6% as Rate
2. Enter 5 as Nper
3. Leave Pmt blank
4. Enter 100 as PV
5. Leave Type blank
6. Note that if you enter PV as a positive value, FV will be negative

FV = PV [(1+r)t] where [(1+r)t] is the future value of $1.00. This is sometimes called the Future Value Interest Factor (FVIF)

**Another example**: Invest $25 for 2 years at 9%. What will it grow to?

FV = PV (1+r)t

 = $25 (1.09)2

 = $25 (1.09) (1.09) = $29.70

**Excel**: FV function

Enter .09 or 9% as Rate

Enter 2 as Nper

Leave Pmt blank

Enter 25 as PV

Leave Type blank

If you want to see FV as a positive value, either enter -25 for PV or solve for -FV

 25 (1.09) 25 (1.09)2

25 27.25 29.70

 0 1 2

Try $10 at 5% for 30 years

FV = PV (1+r)t

 = 10 (1.05)30

 = 10 (4.3219) = $43.22

Note that t can be any time period (month, week, year, quarter, etc.) – it is the number of compounding periods and r is the interest rate **per compounding period**.

**Compounding Period**: How often interest is posted. Immediately after it is posted, you start earning (or paying) interest on the interest.

**Example**: Credit cards: Interest accrues monthly

 Monthly rate = 1.5%

 You charge $100 on your card

 You wait 2 years to pay it off - what do you owe?

FV = PV (1+r)t where r = 1.5% (interest rate per month) and t = 24 months

 = 100 (1.015)24

 = $142.95 = $100 principal plus $42.95 interest

Question: If 1.5% = monthly rate, what is the annual rate?

**APR =** **Annual Percentage Rate:** the most commonly used way to express interest rates.

APR = r · m where r = int. rate per compounding period and m = number of compounding periods in a year.

1.5% / month = 18% APR because (1.5) (12) = 18.

 However, this ignores the compounding that takes place during the year and doesn’t give you the same result at the end of two years that we got earlier.

If we use an annual rate of 18%: FV = 100 (1.18)2

 = $139.24 which is not $142.95

APR is the most commonly used interest rate. Whenever you see an interest rate, you should assume it is the APR unless it is specified otherwise.

APR/m gives you the interest rate per compounding period because APR is found by multiplying the interest rate for the compounding period by m.



This is a general formula where you are compounding m times per year for n years. Since APR = (r) (m), it must be that APR/m = r. So the above formula is equivalent to

FV = PV (1+r)t

Note what happens when you take this to the limit:  as t → ∞, the value approaches 2.718……. the value of e.

**Continuous compounding** - use e = 2.718281828... = approx. 2.718

FV = PV(ert)

Note that if r = 10% and t = 1; e.1 = (2.718).1 = 1.10517 ⇒ 10.517% = EAR

On Excel, use the “exp” function with r·t as the argument to find the FV

To find the continuously compounded rate of return for a given time period, use 



Example: Your investment is worth $100 on March 1 and it is worth $102 on April 1.

The monthly return is 2% if there was monthly compounding. This means that it grew from $100 to $102 at the end of the month (on April 1).

The continuously compounded monthly return was  = .0198 = 1.98%

Note that this assumes your investment was continually growing (from $100 to $102) over the course of the month. It did not remain at $100 till the end of the month and then jump to $102.

How much money will you have at the end of the year?

With monthly compounding, FV = PV (1+r)t = 100 (1.02)12 = 126.82

With continuous compounding, FV = PV(ert) = 100(e(.0198)(12)) = 126.82

What is the FV of $100 in 3 yrs. with 10% APR (stated rate) with the following compoundings:

Annually: 100(1.1)3 = $133.10

Semi-Annually: 100(1.05)6 = $134.01

Monthly: 100(1.0083)36 = $134.82

Continuously: 100[(e)(.1)(3)] = $134.99

We can take the equation: FV = PV (1+r)t and rearrange it to get

PV = FV\_\_\_

 (1+r)t

PV = value today of a future cash flow

r = discount rate (interest rate)

To calculate PV we discount FV at interest rate r over t periods

**Example**: How much to invest today for it to grow to $500 in 2 years if the interest rate is 7%?

PV = \_FV\_\_

 (1+r)t

 = 500 .

 (1.07)2

 = 436.72

Invest $436.72 today at 7% to get $500 in 2 years.

436.72\_\_\_\_\_\_\_\_\_467.29\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_500.00\_\_

 0 1 2

**Another Example**:

What is the value today of $50,000 to be received in 10 years if the interest rate is 9% (APR) and we have monthly compounding?

Excel: Find PV under function wizard

Rate = .09/12

Nper = 10 x 12

Pmt = blank

Fv = 50,000

Type = blank

PV = \_FV\_

 (1+r)t

 = 50,000

 

 = 20,396.87

So far, we have 2 basic equations:

1. FV = PV (1+r)t

2. PV = \_\_FV\_\_

 (1+r)t

How can we solve for “r”?

Take (1) and divide each side by PV and then reverse sides to get (1+r)t = FV

 PV

Now raise each side to the power of 1/t to get 1+r =  This is how we find r.

r =  - 1

Suppose someone tells you if you give him $100 today, he’ll give you $120 in 3 years. Is this a good deal or a bad deal? We must determine what interest rate we are getting.

But note: To solve this, we must make an assumption about how frequently our money is being compounded.

PV = $100 FV = $120 t = 3 years ( if we assume annual compounding)

100 120

 0 1 2 3

r =  - 1

 = - 1

 = (1.2)1/3 - 1

 = .0627 = 6.27%

Using Excel: Find Rate under the function wizard

Nper = 3

Pmt = blank

Pv = -100

Fv = 120

Type = blank

**Solving for t:**

FV = PV (1+r)t

FV = (1+r)t

# PV

ln  = [ln (1+r)] ⋅ t

ln  = t

 ln (1+r)

**Example**:

How long will it take a $1,000 investment to grow to $100,000 if it earns 14% per year with annual compounding?

ln  = 4.60517 / 0.131 = 35.146 years

 ln (1.14) 1,000 100,000

 0 ?

Excel: Find NPER on function wizard

Rate = .14

Pmt = blank

Pv = -1000

Fv = 100000

Type = blank

Note though that if interest is posted at the end of the year, you won’t have it until the end of year 36!

**Summing the Present Values**

You are the agent for a professional athlete. Two contract alternatives are presented to you. Which is better?

These are the cash flows your client is guaranteed to receive:

Contract A: $1 million per year for five years with each payment coming at the end of the year.

Contract B: $3.1 million at the end of the first year and $400,000 at the end of each of the following four years.

Under contract A, your athlete will receive a total of $5 million

Under contract B, your athlete will receive a total of $4.7 million

Unfortunately, you can’t add money received in different time periods - even if they are paired up in the same years like this.

You must get all the money valued at the same time period. We’ll use time zero (today).

If r = 10%

1. $1 mill. = $909,091 B) $3.1 mill = $2,818,182

 (1.1)1 (1.1)1

 $1 mill = $826,446 $400,000 = $330,579

 (1.1)2 (1.1)2

 $1 mill = $751,315 $400,000 = $300,526

 (1.1)3 (1.1)3

 $1 mill = $683,013 $400,000 = $273,205

 (1.1)4 (1.1)4

 $1 mill = $620,921 $400,000 = $248,369

 (1.1)5 (1.1)5 .

 $3,790,786 $3,970,860

So if r = 10%, PVB > PVA

In Excel, you can solve for NPV (*Net* Present Value) to get these answers

**Excel**: Find NPV on function wizard

Rate = .10

Values must be entered as: 1,000,000, 1,000,000, 1,000,000, 1,000,000, 1,000,000

or as 3,100,000, 400,000, 400,000, 400,000, 400,000

If r = 2%

A) $1 mill. = $980,392 B) $3.1 mill = $3,039,216

 (1.02)1 (1.02)1

 $1 mill = $961,169 $400,000 = $384,468

 (1.02)2 (1.02)2

 $1 mill = $942,322 $400,000 = $376,929

 (1.02)3 (1.02)3

 $1 mill = $923,845 $400,000 = $369,538

 (1.02)4 (1.02)4

 $1 mill = $905,731 $400,000 = $362,292

 (1.02)5 (1.02)5 .

 $4,713,460 $4,532,443

So if r = 2% then PVA > PVB

Notice that the higher the interest rate, the more important it is to get money early on.

The main point is that if you wish to compare money received in different time periods, you must first convert all CF to PV.

You can always add PVs but you can never add money in different time periods unless the interest rate is zero.