Asset Pricing Part 2 – Chapt. 11 in RWJJ

Mean/Variance Analysis – Risk-averse investors like a high expected (mean) return and a low standard deviation (variance). They like return and dislike risk.

So if we plot expected return on the vertical axis and standard deviation on the horizontal axis, investors prefer to be as far northwest as possible.

Every asset (and combination of assets) has a specific E(R) and σ, and will occupy a single point in mean-variance space.

Look at stocks A and B from Asset Pricing Part 1 notes.

**A**: E(RA) = 10%; σA = 7.9%

**B**: E(RB) = 8%; σB = 6.1%

As we saw in our Part I notes, 50% in A and 50 % in B gives us E(RP) = 9% and σP = 4.25% (where P stands for Portfolio AB).

Different combinations of A and B (other than 50/50) will yield different E(RP) /σP points. The collection of these points is called the opportunity set.

E(RP) is always a weighted average and σP is always < a weighted average as long as A and B are not perfectly positively correlated.

The covariance (correlation) between A and B determines the shape of the opportunity set. As ρ ↓, the opportunity set bends more to the left, because A and B offset each other more exactly.

At ρ = 1.0, there is no advantage to diversification. σP is just a weighted average of σA and σB.

At ρ = -1.0, there will be some combination of A and B for which σP = 0.

Since risk-averse investors always want more expected return and less standard deviation, the **efficient frontier** is those combinations of A and B that can’t be improved upon - meaning you can’t get ↑ E(RP) with the same σP, or ↓σP with the same E(RP).

Unless we allow for shorting, the efficient frontier starts with the minimum variance portfolio (MVP) and moves up to the highest E(RP) portfolio, which will be 100% invested in the security with the highest E(R).

This works not only with stock A and stock B, but with any two risky assets. Further, it works with any two portfolios of risky assets.

If we like E(R) and dislike σ, we will only hold portfolios on the efficient frontier.

Suppose we look at all risky assets and all portfolios of risky assets.

We still have an efficient frontier that looks like what we showed before (see figure 11.6 in your text).

X (in figure 11.6) is the one stock with the single highest expected return.

MV (in figure 11.6) is the combination of stocks with the lowest standard deviation.

Different people will choose different points on the efficient frontier depending on their individual levels of risk aversion, but everyone will pick some point on the efficient frontier. All other points are **dominated**.

We can use Excel to find the efficient frontier for a set of risky assets if we first calculate their expected returns, standard deviations, and the correlations between the assets. You will do this in future finance courses.

We calculate the variance and standard deviation for a portfolio of N securities the same way we did for a portfolio of 2 securities.

Using the matrix approach, we construct an N x N matrix. Building on the 2 x 2 matrix that we constructed earlier, a 3-asset portfolio will utilize a 3 x 3 matrix, a 4-asset portfolio will utilize a 4 x 4 matrix, etc. No matter the size of the portfolio, the addition of each cell in its weighted variance/covariance matrix gives us the variance of the portfolio.

With an N x N variance/covariance matrix, each diagonal cell will contain a variance term and each off-diagonal cell will contain a covariance term.

For a 3-stock portfolio, we have 9 cells. 3 diagonal cells and 6 off-diagonal cells.

The number of diagonal cells will always equal the number of stocks in the portfolio since each variance is counted once.

The number of off-diagonal (covariance) cells will always equal N2 – N. Or alternatively, N(N – 1).

As N → ∞, the off-diagonal cells dwarf the diagonal cells and the variances lose their importance. What remains important is the covariances.

For an equally-weighted portfolio, the lowest variance you can get is the average covariance of the securities in the portfolio. Adding more securities to the portfolio pushes you closer and closer to that point. This can be seen in Figure 11.7 of your text.

If all investors are risk averse, and by adding more securities to your portfolio you can reduce risk (σP), we can assume that all sensible investors will hold **diversified portfolios**. If they do, a security’s standard deviation is not the appropriate measure of its risk. Instead, it is the security’s **covariance** with other securities in the portfolio because that is the determinant of its contribution to the variance of the portfolio.

We now add the **risk-free asset** to the graph. If there is a risk-free asset (remember that we will use a treasury bill or note as a proxy for the risk-free asset), it will have the same return in all states of the world and a standard deviation of zero (that is the definition of risk-free). This means that **its** **covariance (and its correlation) with any and all other assets must be zero**. Look at the covariance formula and you will see why this is the case.

So, if we plot the risk-free asset on our E(R) / σ graph, it will fall on the vertical axis and must be considered to be part of the efficient frontier (since no combination of risky assets can have a lower σ).

With any two-asset portfolio (including one where one of the assets is the risk-free asset), the **expected return is a weighted average** of the expected returns on each asset.

Example: If the risk free rate Rf is 6% and E(RA) = 10%, a portfolio with 75% of its money invested in A and 25% in Rf will have an expected return of .25(6%) + .75(10%) = 9%

Significantly however, since the risk-free asset has a covariance of zero with every other asset, the **standard deviation** of a two-asset portfolio **where one asset is risk-free** will equal the weight of the risky asset times the standard deviation of the risky asset.

Example: Again, put 75% of your money in asset A and 25% in the risk-free asset.

 σ2A = .0076

 σ2Rf = 0

σ2P = wA2 σA2 + wRf2 σ Rf 2 + 2wAwRf CovA,Rf

 = (.75)2 (.0076) + (.25)2 (**0**) + 2(.75)(.25)(**0**)

 = .004275 + 0 + 0 = .004275

σP = .06538

Note that (.75)2 is the square of the proportion in A. Also, .0076, the variance of A, is, by definition, the square of the standard deviation of A. So if we take the square root of each, we get the **standard deviation of A weighted by the proportion of money invested in A.**

 = σP (a linear equation)

This means that we have a linear (straight-line) relationship between the risk-free asset and any risky asset (or portfolio of risky assets) on our E(R) / σ graph.

Further, if we can **borrow at Rf** just as we can lend at Rf, the line extends out beyond the riskyasset.

**Important Leveraging Example:**

If you invest $100 in A, your expected return is 10% ($110).

If you borrow $100 at 6% and invest it, along with your $100 in A, you expect to have $220 at the end of period 1. After paying back $106 on what you borrowed, you are left with $114 – a 14% return on the $100 that you put up.

You invested 200% of your money in A and -100% of your money in Rf. Your expected return is still a weighted average of the expected returns on the 2 assets.

E(RP) = w1 E(R1) + w2 E(R2) = 2(10%) + -1(6%) = 14%

where you invested 200% of your money in A and -100% in Rf.

If your actual return is only 2%, your $100 becomes $102.

But if you borrow an additional $100 (again, at 6%) to invest, the $200 becomes $204 and you still have to pay back $106. This leaves you with just $98 – worse than you would have been if you hadn’t borrowed.

So “leveraging” your investment increases both expected return and standard deviation.

Once we allow for riskless borrowing and lending, our efficient frontier changes dramatically. You can see this in figure 11.9 in your text.

**Any combination of portfolio** **A (in your text) and Rf dominates** all other portfolios.

Whatever lies at point A in figure 11.9, it must be a very special asset or portfolio of assets and be greatly in demand. In fact, there is no reason to hold any other portfolio of risky assets.

This new efficient frontier is called the **Capital Market Line (CML).**

It can be shown that if all investors are risk-averse and have homogeneous expectations, point A must be the **market portfolio** – a market-value-weighted portfolio consisting of all risky assets.

This brings us to the conclusion that all rational risk-averse investors will hold the market portfolio and will also either borrow or lend at the risk-free rate. How much they borrow or lend determines where on the CML they end up and is determined by their individual level of risk aversion.

If all rational investors hold the market portfolio, the best way to measure the risk of an individual security is **how it affects the standard deviation of the market portfolio**, not the standard deviation for the individual security. Using our earlier analogy, the market portfolio is our team and we are concerned with how each player on the team affects the team’s performance.

A stock’s beta is the way we measure how that stock affects the riskiness (the standard deviation) of a diversified portfolio.

**Beta** - Sensitivity of a stock’s return to the return on the market portfolio.

 - The measure of market risk in a stock

- How much systematic risk a stock has relative to an average stock (or relative to the market).

By definition, βi = Covi,Mkt = ρ \_σi .

 σ2Mkt σMkt

By definition - the market portfolio has a beta of one.

Any stock that moves twice as much (on average) in the same direction as the market portfolio has a beta of 2. Any stock that moves 1/2 as much (on average) in the same direction as the market portfolio has a beta of 0.5. The average beta for all stocks = 1.

**How do we determine a stock’s beta?**

1. Observe monthly rates of return for the stock and the market over the past 5 years.

2. Plot the observations on a graph with the stock’s returns as the dependent (Y) variable.

3. Fit a regression line onto the plot.

4. The slope of the regression line is the stock’s beta.

**Market Risk** - Risk from the market which can’t be diversified. This is the **risk found in the slope of the regression line**. Example: If Intel has a beta of 1.2, this means that if the market is up 1%, Intel is up 1.2% **on average**.

**Unique Risk** - **Risk found in distance of each point from the regression line**. Same Example: If market is up 1%, Intel may be up more or less than 1.2%.

For a portfolio of assets, we assume that the unique risk in those assets will cancel each other out (some will be up when others are down), leaving only the market risk.

**Diversification eliminates unique risk**, but not market risk.

If beta is the change in return caused by the market, what is the **beta of Rf**?

Zero. No change in its return.

Since its covariance with the market is zero, the formula for Beta tells us it must be zero.

**Market Risk Premium** - Risk premium of the market portfolio. We will use a figure of 5.7% (from prior notes). We will consider the S&P 500 to be a proxy for the market.

It is how much we expect the market to outperform the risk-free asset by, on average, over a long period of time.

E(RM – Rf) = the expected reward for bearing market risk (an average amount of risk)

Note that we usually omit the expectation operator and write it as (RM – Rf)

**Beta is the amount of systematic risk born by the investor**. It is the level of risk that cannot be diversified away – and since (according to this logic) every rational investor will hold a well-diversified portfolio, it is the only risk that counts. It is the only risk we are compensated for bearing.

**Risk Premium on Asset i** - Asset i’s beta times the market risk premium.

 βi (RM – Rf)

**Expected Return of Asset i** - Risk-free rate plus the asset’s risk premium

 E (Ri) = Rf + βi (RM – Rf)

Note that a **higher Beta** means a **higher expected return** for your asset and a **higher risk**

E(Ri) is the reward for waiting (risk-free rate) plus the reward for bearing systematic risk times the amount of risk born.

There is no reward for bearing unique risk because it can be easily diversified away.

Since the market portfolio has a beta of 1, when you add a stock with a beta > 1 to the market portfolio, it will increase the standard deviation of the market portfolio (that stock has a higher than average covariance with the market). And when you add a stock with a beta < 1 to the market portfolio, it will decrease the standard deviation of the market portfolio (because it has a lower than average covariance with the market).

**Capital Asset Pricing Model** = CAPM - A theory of the relationship between risk and expected return where risk is measured by Beta.

CAPM says: E (Ri) = Rf + βi (RM – Rf)

**Example**: Green Wave has a beta of 1.06

 Risk-free rate = 3.0%

 Market Risk Premium = 5.7%

What is Green Wave’s Expected Return?

E (RGreenWave) = RF + βi (RM – Rf)

 = 3.0% + 1.06 (5.7%)

 = 9.04%

**Another Example**:

Freeman Industries Beta = 1.34

Dividend paid yesterday = $2.00

Risk-free Rate = 3.0%

Starting today, you expect Freeman to grow at 5%/yr. forever

Market Risk Premium = 5.7%

What should the stock price of Freeman Industries be?

E (Ri) = Rf + βi (RM – Rf)

 = 3.0% + 1.34 (5.7%)

 = 10.64%

So investors require an expected return of 10.64% in order to invest in Freeman Industries. They will then price Freeman so that if its future dividends are paid as currently expected, they will achieve this 10.64% return.

P0 = Div1 = Div0 (1+g)

 r-g r - g

 = $2 (1.05)\_\_\_\_ = 2.1\_\_\_ = $37.23

 .1064 - .05 .0564

**Important Assumptions we make when we use the CAPM to price assets as described above:**

1. The beta that I calculate based on a stock’s covariance with the market in the past will carry forward into the future
2. I know how much expected return investors require as compensation for bearing the risk of the market (the market risk premium)
3. All assets are held as part of a well-diversified portfolio – technically, we assume that everyone holds the market portfolio, but a well-diversified portfolio will suffice
4. Investors are risk-averse and are only concerned with expected returns and standard deviation

These are HUGE assumptions. Should we make them? What if they aren’t valid? These (and other) questions will be addressed in future finance courses.

**Important Points behind the CAPM:**

1. Investors are only concerned with Market Risk since unique risk can be diversified away.

2. Investors require extra expected return for taking on extra market risk.

3. There is no extra expected return as compensation for taking on unique risk since it can be diversified away.

The CAPM isn’t perfect. In fact, there are some very significant concerns about its validity. Your text discusses some of these concerns and some alternatives to the CAPM.

Still, CAPM is the most widely-used asset-pricing model and clearly shows that there is a **reward for bearing risk** and **a benefit from diversification**.